APPLICATION OF THE EIGEN-EMITTANCE CONCEPT TO DESIGN ULTRA-BRIGHT ELECTRON BEAMS*

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Abstract

Using correlations at the cathode to tailor a beam's eigen-emittances is a recent concept made useful by the symplectic nature of Hamiltonian systems such as beams in accelerators. While introducing correlations does not change the overall 6-dimensional phase space volume, it can change the partitioning of this volume into the longitudinal and two transverse emittances, which become these eigen-emittances if all the initial correlations are unwound and removed. In principle, this technique can be used to generate beams with highly asymmetric emittances, such as those needed for the next generation of very hard X-ray free-electron lasers. This approach is based on linear correlations, and its applicability will be limited by the magnitude of nonlinear effects in photoinjectors which will lead to mixing in phase space that cannot be unwound downstream. Here, we review the eigen-emittance concept and present a linear eigen-emittance design leading to a highly partitioned, and transverse ultra-bright, electron beam. We also present numerical tools to examine the evolution of the eigen-emittances in realistic accelerator structures and results indicating how much partitioning is practical.

INTRODUCTION

X-ray free electron lasers (XFELs), such as Los Alamos' Matter and Radiation in Extremes (MaRIE) project [1] require transversely bright electron beams. One approach to achieving the necessary low emittance in a dimension is via emittance partitioning, in which a large emittance may be partially transferred to a different dimension, using appropriate optics. This has been demonstrated in the flat-beam transform [2, 3], which exchanges emittance between transverse dimensions, and in tranverse-to-longitudinal emittance exchange [4]. The general case of emittance exchange between any two dimensions is discussed by Carlsten et. al. [5].

Motivated by these ideas, we investigate making two transverse emittances small at the expense of the longitudinal emittance, thus satisfying the emittance requirements for next generation XFELs. Our goal is use the eigen-emittance concept [6] to achieve two very small emittance values. The beam eigen-emittance values are conserved during linear beam transport, after the beam is initially generated. For an uncorrelated beam, they coincide with the three beam emittances. If correlations are introduced when the beam is generated at the cathode such that two of the eigen-emittances are very small, it should be possible to remove these correlations and recover the eigen-emittances as the emittance values, provided nonlinear effects are not too large.

We search for cases with two small eigen-emittances when the minimum number of correlations that may produce this case are present in the electron bunch. As this is the least complicated scenario to produce two small eigen-emittances, it will require the least optics to remove the correlations from the beam. This report of our investigation is structured as follows. Firstly, we outline the theory required to study correlations and their resulting eigen-emittance values. This theory is then applied numerically to investigate which combinations of correlations result in two small eigen-emittance values. Finally, we discuss the possibility of implementing these schemes to obtain transversely bright electron beams.

BACKGROUND THEORY

In this section, we outline the coordinate system used, and briefly describe the eigen-emittance concept and how correlations can be treated theoretically.

We wish to make the two transverse emittances small at the expense of the longitudinal emittance, so we work with the full 6-dimensional phase-space. We use canonical coordinates, $\mathbf{s} = (x, p_x, y, p_y, z, p_z)$, where p_x, p_y and p_z are the canonically conjugate momenta to the configuration space coordinates, x, y and z, respectively. We use dimensionless coordinates which are the deviations from a reference trajectory, $\mathbf{s}_t$, defined in the same manner as in Carlsten et. al. [5], i.e.,

$$\begin{aligned} (x-x_t)/l &\mapsto x, & (p_x-p_{xt})/\delta &\mapsto p_x, \\ (y-y_t)/l &\mapsto y, & (p_y-p_{yt})/\delta &\mapsto p_y, \\ (z-z_t)/l &\mapsto z, & (p_z-p_{zt})/\delta &\mapsto p_z, \end{aligned} \tag{1}$$

where l and δ are scaling factors for the position and momentum coordinates, respectively. We leave these scale factors undefined, as we are simply interested in how the eigen-emittance values change as correlations are increased. The conversion from canonical coordinates to more common coordinates, such as time and energy, are discussed in detail by Carlsten et. al. [5].

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The Hamiltonian motion of a beam has three conserved moments, which can be chosen as the quantities known as the eigen-emittances. These eigen-emittances coincide with the beam emittances when no correlations are present. In practice, these quantities can simply be obtained from the beam matrix as the absolute value of the eigenvalues, λ_j, of the characteristic equation, $det(J\Sigma - i\lambda_j I) = 0$, where I is the identity matrix and the only non-zero entries in the matrix, J, are the 2×2 block diagonal entries containing the skew-symmetric matrix,

$$J_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} . \tag{2}$$

Both I and J have the same dimensionality as the phase-space. A more detailed discussion of eigen-emittances is provided by Dragt, Neri and Rangarajan [6].

We use the same approach described in Yampolsky et. al. [7], extended to six dimensions. Correlations are introduced via a matrix of the form,

$$C = \begin{pmatrix} 0 & 0 & c_{13} & c_{14} & c_{15} & c_{16} \\ 0 & 0 & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & 0 & 0 & c_{35} & c_{36} \\ c_{41} & c_{42} & 0 & 0 & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & 0 & 0 \\ c_{61} & c_{62} & c_{63} & c_{64} & 0 & 0 \end{pmatrix} , \tag{3}$$

which we refer to as the *C-matrix*. Beginning with an initially uncorrelated beam, with beam matrix, Σ_0, general correlations between can be introduced using

$$\Sigma = (I + C)\Sigma_0(I + C)^T . \tag{4}$$

As we wish to obtain two small eigen-emittances, a minimum of two entries in the C-matrix will be required. These two correlations must also couple all three (spatial) dimensions.

Two individual correlations in the C-matrix can be introduced via

$$\begin{aligned} \Sigma &= (I + C_2)(I + C_1)\Sigma_0(I + C_1)^T(I + C_2)^T \\ &\equiv (I + C)\Sigma_0(I + C)^T \end{aligned} \tag{5}$$

The order in which these are introduced may be important. If C_1 and C_2 commute, then the correlations are independent and the order is not important. If if C_1 and C_2 do not commute, the order in which they are introduced determines whether they are dependent or independent. When two correlations are independent, the C-matrix contains two non-zero entries. When they are dependent, the number of non-zero entries in the C-matrix is three.

We discuss the results for a numerical analysis of this approach in the following section.

RESULTS

We search for pairs of correlations that lead to two small eigen-emittance values and one large. To do so, we numerically vary the appropriate entries in the C-matrix using Mathematica and examine the resulting eigen-emittances. For dependent correlations, the final C-matrix is calculated using Eq. (5). The number of cases to be evaluated can be reduced by recognizing symmetries in the problem, such as the cyclic permutations of the coordinates. For numerical purposes, we begin with initial beam emittances 0.7/0.7/1.4 for the $x/y/z$ emittances, which are equivalent to the eigen-emittances in the initially uncorrelated beam. When we discuss specific correlations, the final variable in the correlated beam is the first in the pair and the initial variable that introduces the correlation is the second, e.g. if only an x-y correlation is used, the functional dependence is $x(x_0, y_0)$.

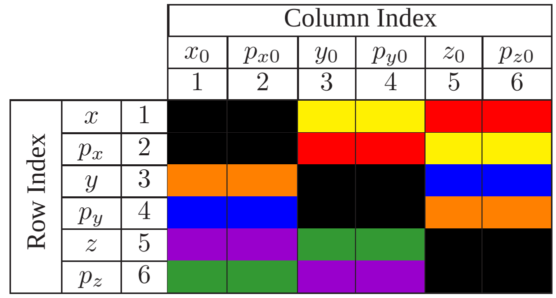

Figure 1: Color chart of independent correlations leading to two small eigen-emittances. Two entries in the C-matrix need to be chosen, one from each block of the same color. Black entries are not considered, as they do not correlate two of the three dimensions. The column indices correspond to the initial coordinates of the uncorrelated beam and the row indices are the coordinates after the correlations have been introduced.

We have found that it is possible to obtain two small eigen-emittances for all dependent correlations and for the combinations of independent correlations given by the color chart of Fig. 1. The independent correlations which lead to two small eigen-emittances are those in which the coordinates of the correlated beam involved in introducing the correlations are canonically conjugate. The way in which the eigen-emittances vary with increasing correlations is illustrated in Fig. 2 for a case which leads to two small eigen-emittance values.

We have found a number of cases that lead to our desired result of two small eigen-emittances. We discuss the possibilities of implementing these theoretical cases in the following section.

DISCUSSION

We have found that combinations of two correlations can lead to two small eigen-emittance values, however, it is not as simple to find a scheme that may be physically realized. While we can theoretically discuss correlations depending on the beam momenta, no practical implementation currently exists. Angular momentum, in the form of p_x-y and p_y-x correlations, is simple to introduce to a beam using a solenoid providing magnetic field on the cathode, however

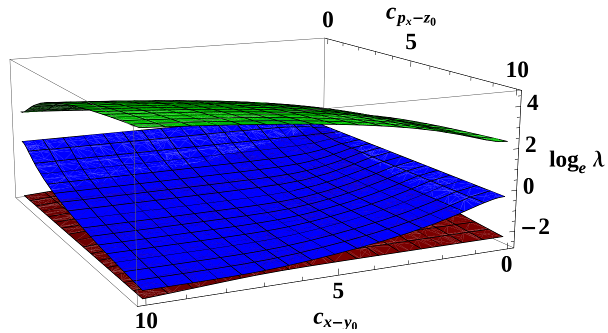

Figure 2: x-y and p_x-z correlations are an example of a case resulting in two small and one large eigen-emittance. The variation of eigen-emittance values with increasing correlation are shown here.

it is difficult to imagine a scheme in which one correlation is introduced without the other and these two correlations do not lead to two small eigen-emittances by themselves. Initial investigation of combinations containing both these correlations and a third indicate that they do not lead to two small eigen-emittances, and support the findings of Yampolsky et. al. [7] which studied these correlations combined with what is effectively a z-x correlation. Additionally, a p_y-z or p_x-z correlation would be difficult to create at the cathode.

Given the above, the only correlations that might be physically realizable are the purple and green blocks of Fig. 1. A p_z-x or p_z-y correlation could be obtained by scanning a drive laser with frequency modulation across a photocathode or using a photocathode with work function variation. This will also introduce z-x or z-y correlations, respectively, however an initial study of more than two correlations shows that it is still possible to obtain two small eigen-emittances with these correlations present. The respective z-y or z-x correlations that are needed to produce two small eigen-emittances could be produced using a drive laser with a tilted pulse front, as in the scheme described in Yampolsky et. al. [7] or a photo-cathode recessed at an angle with changing work function across the surface, such as suggested in Carlsten et. al. [5].

The above discussion shows that a minimal independent correlation scenario is difficult to implement, but a possibility is using p_z-x/y with z-y/x correlations. This leaves the possibility of the dependent correlations. We have not investigated adding additional correlations to two independent correlations, so we limit the discussion to cases where we only introduce two dependent correlations. Based on the previous discussion, single correlations that we can reasonably expect to introduce are correlations between the coordinate variables and p_z correlations that depend on one of the transverse coordinates. Of these, a number of combinations could be possible. One that may be simple to implement could be a laser pulse with a phase-front tilt with an elliptical cathode, giving z-x and x-y correlations or z-y and y-x correlations.

A remaining question is whether a large enough correlation can be achieved to produce a small enough eigen-emittance value. Transverse emittances for the MaRIE XFEL need to be 0.15 μm or less, while a longitudinal emittance of up to 180 μm is acceptable [5]. Whether the necessary eigen-emittance values can be achieved with acceptable correlations that do not lead to extreme aspect ratios in the beam is yet to be investigated. Additionally, if these small eigen-emittance values can be achieved, it is important that nonlinear effects are not large enough to significantly alter the eigen-emittance values.

We have identified cases with promise to lead to two small eigen-emittance values using numerical tools. Further investigation into these cases is warranted, to make sure they are able to be implemented practically.

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